Estimating Key Parameters of a CES Production Function in a Regional Environmentally Extended CGE Model Framework: a GME Approach

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Introduction

• Computable General Equilibrium (CGE) Models calibration procedure is carried out on a database formed by Social Accounting Matrices (SAM) as the macro-accounting core, which represents the economic situation one aims at modelling.

• To calibrate the CGE model we need other supporting information such as satellite accounts and the estimation of key parameters of the functions used to model behaviours in production and consumption spheres need times series and cross section data.

•Estimation of such elasticity is one of the crucial points of the calibration procedure

•Mansur and Whalley (1981) use times series to estimate elasticities in both consumption and production sphere in calibrating a CGE model for taxation purposes.

•Arndt, Robinson and Tarp (2002) performed a Maximum Entropy estimation using times series data joint to prior information on elasticity to estimate trade elasticity associated to a CES aggregate function in a CGE for Mozambique.

•When times series or cross section data are not available elasticities cannot be directly statistically estimated and are taken from external sources, that is the elasticity are imputed with figures taken from similar contexts, through a sort of average or central tendency of estimates from a literature survey.

•This solution is partly non-statistical because of non availability of data.

•In this paper we propose to adapt a generalized maximum entropy approach using the information contained in the RESAM only to estimate the substitution elasticity as regards the production sphere .

Estimating a CES Value Added/Intermediate Consumption Substitution Elasticity Through the GME Approach in a RECGE Framework

The GME Approach

The estimation of a parameters of a model **Y=Xβ+u** based on a RESAM in a RECGE framework is a tipical ill-posed problem.

GME approach, maximizes the joint entropies (dual loss objective function) of both signal and noise.

$$F(\delta) = F(\beta) + F(\mathbf{u}) = F(\beta) + F(\mathbf{y}-\mathbf{X}\beta) = H(\mathbf{p}) + H(\mathbf{w}) = -\sum_{i=1}^{n} p_i \log(p_i) - \sum_{i=1}^{n} w_i \log(w_i)$$

Subject to $\mathbf{y} = \mathbf{XZp} + \mathbf{Vw}$ and $\mathbf{1_N}=(\mathbf{I_N}\otimes\mathbf{1'_D})\mathbf{p}$ and $\mathbf{1_N}=(\mathbf{I_N}\otimes\mathbf{1'_H})\mathbf{w}$

Where δ = [β,u] the convex set B* = BV and V is the convex hull of the state space of u, Z is the support space of p which is the probability associated to β, w is the probability associated to u and H(p) and H(w) are the entropy measures for the signal and noise respectively and are defined over the joint space B* such that β = Ep (Z) and u = Ew.(V).

CES Value Added/Intermediate Consumption Substitution Estimation

$$\log Y_i = \log \alpha_i - \frac{1}{\rho_i} \log\left[\delta_i X_{1,i}{}^{-\rho_i} + (1-\delta_i) X_{2,i}{}^{-\rho_i}\right] + \varepsilon_i \qquad i=1,\ldots,N$$

- Y_i is the total output
- $X_{1,i}$ is the Value Added
- $X_{2,i}$ is the Intermediate Consumption
- ρ_i is the substitution parameter from which the elasticy $\sigma_i=1/1+\rho_i$ is derived
- α is the efficiency parameter
- δ is the share parameter

The GME estimator maximizes the dual loss objective function of all the probabilities representing the signal ρ, α, δ and the noise ε:

$$H(\mathbf{q},\mathbf{p},\mathbf{b},\mathbf{w}) = -\sum_{i=1}^{N}\sum_{d=1}^{D} q_{i,d}\log q_{i,d} - \sum_{i=1}^{N}\sum_{s=1}^{S} p_{i,s}\log p_{i,s} - \sum_{i=1}^{N}\sum_{m=1}^{M} b_{i,m}\log b_{i,m} - \sum_{i=1}^{N}\sum_{h=1}^{H} w_{i,h}\log w_{i,h}$$

Subject to:

$$\log Y_i = \log\left(\sum_{s=1}^{S} a_{i,s}p_{i,s}\right) - \frac{1}{\sum_{d=1}^{D} z_{i,d}q_{i,d}}\log\left[\left(\sum_{m=1}^{M} t_{i,m}b_{i,m}\right)X_{1,i}^{-\sum_{d=1}^{D} z_{i,d}q_{i,d}} + \left(1-\sum_{m=1}^{M} t_{i,m}b_{i,m}\right)X_{2,i}^{-\sum_{d=1}^{D} z_{i,d}q_{i,d}}\right] + \sum_{h=1}^{H} v_{i,h}w_{i,h}$$

$$\sum_{d=1}^{D} q_d = \sum_{s=1}^{S} p_s = \sum_{m=1}^{M} b_t = \sum_{h=1}^{H} w_h = 1$$

CES re-parametrization

$$\log Y_i = \log\left(\sum_{s=1}^{S} a_{i,s}\, p_{i,s}\right) - \frac{1}{\sum_{d=1}^{D} z_{i,d}\, q_{i,d}} \log\left[\left(\sum_{m=1}^{M} t_{i,m}\, b_{i,m}\right) X_{1,i}^{-\sum_{d=1}^{D} z_{i,d} q_{i,d}} + \left(1-\sum_{m=1}^{M} t_{i,m}\, b_{i,m}\right) X_{2,i}^{-\sum_{d=1}^{D} z_{i,d} q_{i,d}}\right] + \sum_{h=1}^{H} v_{i,h}\, w_{i,h}$$

$$\alpha_i = \sum_{s=1}^{S} a_{i,s}\, p_{i,s} = \begin{bmatrix} \mathbf{a}_1' & 0. & . & 0 \\ 0 & \mathbf{a}'_2 & . & . \\ . & . & . & . \\ 0 & . & . & \mathbf{a}'_N \end{bmatrix} \begin{bmatrix} \mathbf{p}_1 \\ \mathbf{p}_2 \\ . \\ \mathbf{p}_N \end{bmatrix}$$

$$\delta_i = \sum_{m=1}^{M} t_{i,m}\, b_{i,m} = \begin{bmatrix} \mathbf{t}_1' & 0. & . & 0 \\ 0 & \mathbf{t}'_2 & . & . \\ . & . & . & . \\ 0 & . & . & \mathbf{t}'_N \end{bmatrix} \begin{bmatrix} \mathbf{b}_1 \\ \mathbf{b}_2 \\ . \\ \mathbf{b}_N \end{bmatrix}$$

$$\varepsilon_i = \sum_{h=1}^{H} v_{i,h}\, w_{i,h} = \begin{bmatrix} \mathbf{v}_1' & 0. & . & 0 \\ 0 & \mathbf{v}'_2 & . & . \\ . & . & . & . \\ 0 & . & . & \mathbf{v}'_N \end{bmatrix} \begin{bmatrix} \mathbf{w}_1 \\ \mathbf{w}_2 \\ . \\ \mathbf{w}_N \end{bmatrix}$$

$$\rho_i = \sum_{d=1}^{D} z_{i,d}\, q_{i,d} = \begin{bmatrix} \mathbf{z}_1' & 0. & . & 0 \\ 0 & \mathbf{z}'_2 & . & . \\ . & . & . & . \\ 0 & . & . & \mathbf{z}'_N \end{bmatrix} \begin{bmatrix} \mathbf{q}_1 \\ \mathbf{q}_2 \\ . \\ \mathbf{q}_N \end{bmatrix}$$

The solution to our maximization problem is obtained by solving a lagrangean maximisation where the fist order condition yields the optimum solution for $\hat{q}$, $\hat{p}$ $\hat{b}$ $\hat{w}$ which represent the posterior distribution on the support that satisfies the observations and are the closest to the prior distribution.

Substituting the optimal solution for $\hat{q}$ $\hat{b}$ $\hat{p}$ $\hat{w}$ we can derive the point estimate for the parameters:

$$\hat{\rho}_i^{GME} = \sum_{d=1}^{D} z_{i,d}\hat{q}_{i,d} \qquad \hat{\delta}_i^{GME} = \sum_{m=1}^{M} t_{i,m}\hat{b}_{i,m}$$

$$\hat{\alpha}_i^{GME} = \sum_{s=1}^{S} a_{i,s}\hat{p}_{i,s} \qquad \hat{\varepsilon}_i^{GME} = \sum_{h=1}^{H} v_{i,h}\hat{w}_{i,h}$$

RESAM-CES Data Description

Table1: Sardinia RESAM

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51
	AAGR	AMINE	AFOOD	ACOKE	ATRANSP	AOMAN	AENERGY	ACONSTR	ARETAIL	ABUS	AOTH	AUWASTE	ASWASTE	ADWASTE	CAGR	CMINE	CFOOD	CCOKE	CTRANSP	COMAN	CENERGY	CCONSTR	CRETAIL	CBUS	COTH	CUWASTE	CSWASTE	CDWASTE	LABINC	SEINC	CAPINC	IMPSOCON	SSOCONTI	AWP	H-9300	H-9300-124	H-12400-15	H-15500-24	H-24800-31	H-31000	FIRMS	GOV	PRODTAX	PRODSUB	VAT	S-I	EINV	EXPTAX	Italy	World	TOTAL
1															1508																																				1508
2																276																																			276
3																	2263																																		2263
4																		3877																																	3877
5																			1236																																1236
6																				3541																															3541
7																					1817																														1817
8																						3736																													3736
9																							11066																												11066
10																								7437																											7437
11																									11748																										11748
12																										77																									77
13																											238																								238
14																												9																							9
15	175	0	464	5	0	16	1	1	47	5	16	0	0	0																					53	66	57	97	27	43	0	1				1	0		797	10	1882
16	0	23	2	1023	1	63	197	25	0	1	1	0	0	0																					0	0	0	0	0	0	0	0				14	0		167	20	1537
17	122	0	355	28	0	3	0	0	367	5	33	0	0	0																					280	347	294	507	142	224	0	5				0	0		945	145	3802
18	68	16	14	711	20	198	121	55	371	83	466	3	8	0																					78	135	110	207	72	78	0	0				0	0		968	1530	5312
19	4	9	13	23	227	43	25	131	315	70	155	3	11	0																					74	129	141	261	104	139	0	0				1971	23		459	84	4414
20	11	25	107	127	244	1074	16	909	310	150	148	5	15	0																					156	282	275	514	196	312	0	0				301	0		1560	308	7045
21	33	19	41	111	17	197	267	25	242	62	145	0	1	0																					105	122	118	178	59	83	0	5				0	0		37	0	1867
22	0	1	3	7	4	17	120	414	62	38	208	1	2	0																					4	4	4	6	3	4	0	0				2845	0		20	2	3769
23	84	26	273	276	112	365	46	192	1144	405	355	5	17	0																					638	948	1303	2283	849	960	0	27				494	0		258	596	11656
24	81	25	123	170	117	304	72	291	1517	2002	1272	18	55	4																					86	160	153	397	120	210	0	360				352	0		192	22	8103
25	1	8	8	16	12	33	7	26	408	286	624	4	12	0																					479	523	556	794	360	427	0	6948				103	0		299	54	11988
26	0	0	0	0	0	0	0	0	0	0	0	0	0	0																					5	10	9	13	8	13	0	19				2	0		0	1	80
27	0	0	3	123	0	48	40	0	12	0	0	0	16	0																					0	0	0	0	0	0	0	0				0	0		0	0	242
28	0	0	0	3	0	3	0	0	1	1	1	0	0	0																					0	0	0	0	0	0	0	0				0	0		0	0	9
29	208	66	122	159	135	427	149	434	1928	1031	3535	14	43	2																																					8253
30	288	0	97	3	39	179	0	272	1194	225	529	2	6	0																																					2834
31	168	23	20	215	85	196	229	428	1787	1494	2838	3	10	0																																					7496
32	20	29	48	59	46	155	69	156	502	304	1377	3	9	0																																					2777
33	65	0	10	0	4	21	0	35	134	30	49	0	1	0																																					349
34	0	0	-1	-122	0	-50	-39	1	-2	45	32	10	15	3																																					-108
35																													855	0	859	288	0	0	14	0	0	0	0	0	116	756								10	2898
36																													938	337	1112	316	42	0	0	33	0	0	0	0	267	1737								24	4806
37																													435	1099	889	147	136	0	0	0	14	0	0	0	117	759								9	3605
38																													3271	535	1822	1102	66	0	0	0	0	23	0	0	189	1230								17	8255
39																													1116	230	542	376	28	0	0	0	0	0	7	0	58	379								5	2741
40																													1638	634	715	548	78	0	0	0	0	0	0	6	52	336								5	4012
41																													0	0	890	0	0	0	82	90	42	315	108	158	179	44								64	1972
42																													0	0	667	0	0	-108	339	330	35	1232	414	571	885	2719	1510	-2563	1712	7578		125		21	15467
43	7	3	294	653	5	6	110	43	2	373	3	3	8	0																																					1510
44	-103	-134	-41	-2	-15	-196	-21	0	-735	0	-1316	0	0	0																																					-2563
45	15	0	130	136	68	180	84	195	663	204	30	1	3	0																																					1709
46	258	137	164	131	48	241	324	103	797	623	1247	2	6	0																					499	1619	490	1403	264	770	47	0					-23		1727	2785	13662
47	0	0	0	0	0	0	0	0	0	0	0	0	0	0																																					0
48	3	0	14	22	67	18	0	0	0	0	0	0	0	0																																					124
49															204	206	1158	781	1434	2586	33	21	280	566	155	1	2	0																							7427
50															170	1055	384	654	1744	909	17	14	310	96	92	1	2	0							7	9	3	26	9	13	62	139									5716
51	1508	276	2263	3877	1236	3541	1817	3736	11066	7437	11748	77	238	9	1882	1537	3805	5312	4414	7036	1867	3771	11656	8099	11995	79	242	9	8253	2835	7496	2777	350	-108	2899	4807	3604	8256	2742	4011	1972	15464	1510	-2563	1712	13661	0	125	7429	5712	

Activities			Commodities	
AAGR	Agricoltural, fishes		CAGR	Agricoltural, fishes
AMINE	Mining		CMINE	Mining
AFOOD	Food, drink and tabacco		CFOOD	Food, drink and tabacco
ACOKE	Coke, Petroleum and Chemical		CCOKE	Coke, Petroleum and Chemical
ATRANSP	Machinery and equipment transport		CTRANSP	Machinery and equipment transport
AOMAN	Other manufacturing		COMAN	Other manufacturing
AENERGY	Electricity, gas		CENERGY	Electricity, gas
ACONSTR	Construction		CCONSTR	Construction
ARETAIL	Wholesale, retalil, hotels and catering		CRETAIL	Wholesale, retalil, hotels and catering
ABUS	Financial Intermediation, R&D and other business activities		CBUS	Financial Intermediation, R&D and other business activities
AOTH	Other activities		COTH	Other activities
AUWASTE	Urban waste disposal		CUWASTE	Urban waste disposal
ASWASTE	Special waste disposal	ENV	CSWASTE	Special waste disposal
ADWASTE	Special danderous waste disposal		CDWASTE	Special danderous waste disposal

Results

Table 2: GME parameter estimates

	rho	alpha	delta	sigma
AGR	1.416	1.585	0.594	0.414
MINE	1.896	1.513	0.603	0.345
FOOD	1.544	0.781	0.620	0.393
COKE	1.524	0.938	0.623	0.396
TRANSP	1.547	1.498	0.615	0.393
OMAN	1.302	1.679	0.620	0.434
ENERGY	1.48	1.477	0.615	0.403
CONSTR	1.256	1.946	0.611	0.443
RETAIL	1.076	1.741	0.592	0.482
BUS	1.145	1.633	0.600	0.466
OTH	1.08	1.730	0.555	0.481
ENV	1.872	1.500	0.605	0.348

Table 3: Goodness of fit

	rho R*=1-S($\hat{q}$)	alpha R*=1-S($\hat{p}$)	delta R*=1-S($\hat{b}$)
AGR	0.896	0.858	0.884
MINE	0.920	0.949	0.930
FOOD	0.890	0.780	0.878
COKE	0.882	0.735	0.874
TRANSP	0.899	0.876	0.846
OMAN	0.884	0.900	0.870
ENERGY	0.893	0.880	0.895
CONSTR	0.883	0.919	0.883
RETAIL	0.867	0.975	0.879
BUS	0.873	0.982	0.875
OTH	0.867	0.969	0.899
ENV	0.918	0.829	0.927

Table 4: Parameter support vectors

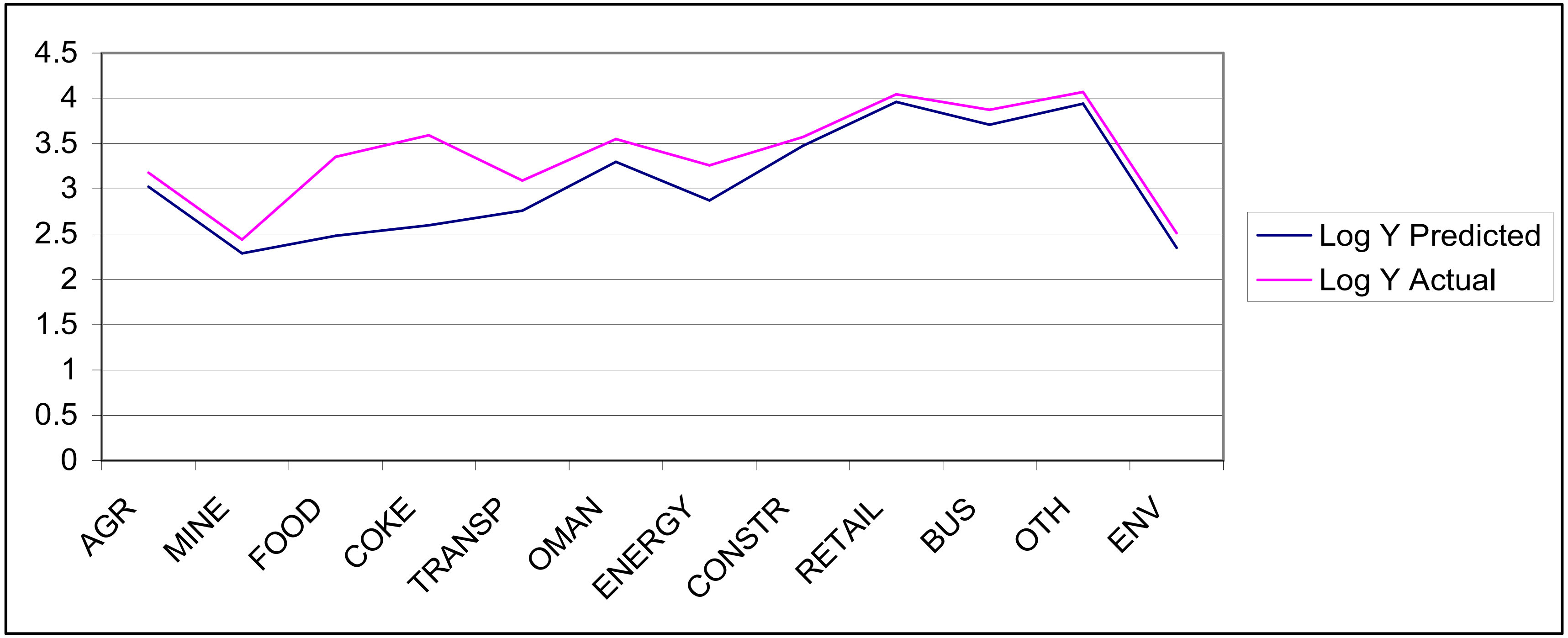
	Parameter support vectors
rho	[1 1.60 2.20 2.80 3.4]
alpha	[0.5 1 1.5 2 2.5]
delta	[0.2 0.4 0.6 0.8 1]

$$Z = \begin{bmatrix} \mathbf{z}_1' & 0. & . & 0 \\ 0 & \mathbf{z}'_2 & . & . \\ . & . & . & . \\ 0 & . & . & \mathbf{z}'_N \end{bmatrix}$$

$$T = \begin{bmatrix} \mathbf{t}_1' & 0. & . & 0 \\ 0 & \mathbf{t}'_2 & . & . \\ . & . & . & . \\ 0 & . & . & \mathbf{t}'_N \end{bmatrix}$$

$$A = \begin{bmatrix} \mathbf{a}_1' & 0. & . & 0 \\ 0 & \mathbf{a}'_2 & . & . \\ . & . & . & . \\ 0 & . & . & \mathbf{a}'_N \end{bmatrix}$$

Figure 1: Total output



Concluding Remarks

- From a statistical point of view the GME estimates of the CES key parameters globally present a good fit.
- From an economic point of view the parameter estimates conforms to the expectation and seem consistent with the Sardinian economic system.
- It is confirmed the possibility of using the RESAM information only to estimate a the parameters of a CES through the GME approach.
- In future it would be interesting to exploit the GME estimation procedure also for the demand sphere.

Shannon Entropy

Suppose that the event E will occur with a certain probability p.
What is the information that the event E will occur?

- If *p* is high, event occurrence has little information.
- If *p* is low, event occurrence is a surprise, and contains a lot of information.

$$H(p) = \log(1/p)$$

$$IF \quad p = 1 \quad then \quad h(p) = 0$$

$$IF \quad p = 0 \quad then \quad h(p) = \infty$$

Normalised Entropy

$$S(\hat{q}) = \left(-\sum_{d} \hat{q}_d \log \hat{q}_d \right) \Big/ D \log(D)$$

$$S(\hat{q}) \in [0,1]$$