The validation of Credit Rating and Scoring Models

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Outline

1. The validation process
2. Literature review
   - Cumulative Accuracy Profile Curve
   - Receiver Operating Characteristic Curve
3. Methodological proposals
   - Curve of Classification Error Costs and Error Costs

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Preliminary definitions

- Credit **rating** and **scoring** models estimate the credit obligor's *worthiness* and provide an assessment of the obligor's future status.
- The **discriminatory power** of a rating or scoring model denotes its ability to discriminate *ex ante* between defaulting and non-defaulting borrowers.
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Each borrower is characterized by two random variables:

- the score S assigned to the borrower is a continuous r. v. with support $(-\infty, \infty)$
- the Bernoulli r.v. B represents the borrower's state at the end of a fixed time-period

$$B = \begin{cases} 1, & \text{the borrower's state is default } (d); \\ 0, & \text{the borrower's state is non default } (n). \end{cases}$$

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The distribution function of the score S is

$$F(s) = pF_d(s) + (1-p)F_n(s)$$

where p is the probability of default $p = P[B = d]$.

The *accuracy* (AC) is

$$AC = pF_d(s) + (1-p)[1 - F_n(s)] = 2pF_d(s) - F(s) + (1-p)$$

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Predicted default (below s)	True Default (TD)	False Default (FD) *Type II error*
Predicted non default (above s)	False Non Default (FN) *Type I error*	True Non Default (TN)
	N_d	N_n

- hit rate $\hat{F}_d(s) = \frac{TD}{N_d}$
- false alarm rate $\hat{F}_n(s) = \frac{FD}{N_n}$
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  where $\hat{p} = \frac{N_d}{N_d+N_n}$

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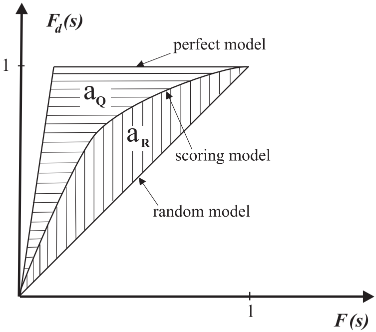
Cumulative Accuracy Profile (CAP) Curve and Accuracy Ratio (AR)



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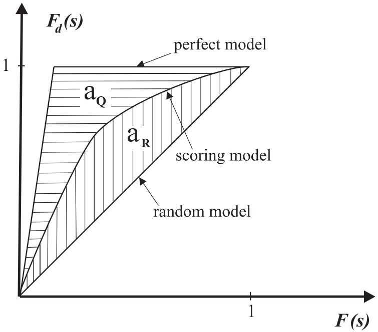

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Cumulative Accuracy Profile (CAP) Curve and Accuracy Ratio (AR)

- *synthetic index* (BCBS, 2005):

$$AR = \frac{a_R}{a_R + a_Q} \qquad AR \in [0, 1]$$

- *optimal cut-off score* (Hong, 2009): the intersection of the CAP curve and the iso-performance tangent line

$$F_d(s) = \frac{1}{2p}\left[F(s) + AC + p - 1\right]$$

- *drawbacks*:
  - dependence on the sample relative frequency of defaulted borrowers;
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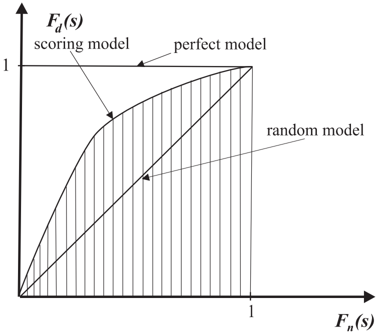

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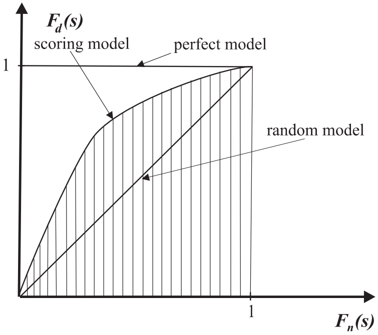

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$$AUC = \int_0^1 F_d[F_n(s)]dF_n(s) \qquad AUC \in [0.5, 1]$$

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Curve of Classification Error Costs (CEC) and Error Costs (EC)

- *Curve*:

$$C[u] = \frac{C_{FN}}{2}\{1 - F_d[F^{-1}(u)]\} + \frac{C_{FD}}{2} F_n[F^{-1}(u)] \quad u \in (0,1)$$

- *Synthetic index*:

$$EC^* = \int_0^1 C[F(s)]dF(s)$$

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- *Point measure* (Zenga, 2007): $U(c) = \frac{\bar{\mu}(c)}{\overset{+}{\mu}(c)}$

where $c \in (-\infty, +\infty)$ and

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